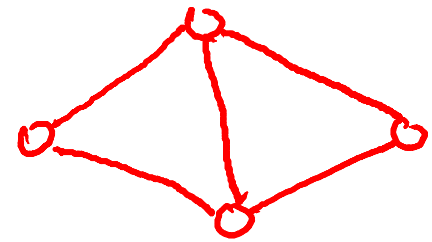
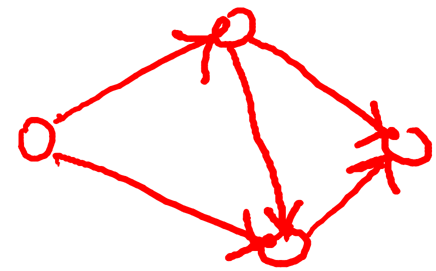
Design and Analysis of Algorithms I

Graph Algorithms

Representing Graphs

Graphs





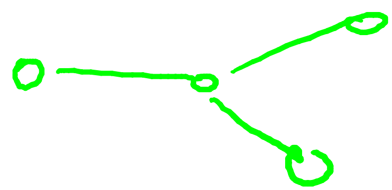
<u>Two ingredients</u>

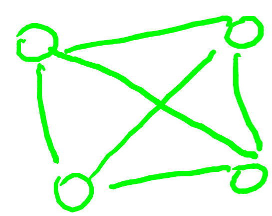
- <u>vertices</u> aka <u>nodes</u> (V)
- <u>edges</u> (E) = pairs of vertices
  - can be <u>undirected</u> [unordered pair]
    or <u>directed</u> [ordered pair] (aka <u>arcs</u>)

<u>Examples</u>: road networks, the Web, social networks, precedence constraints, etc.

Consider an undirected graph that has n vertices, no parallel edges, and is connected (i.e., "in one piece"). What is the minimum and maximum number of edges that the graph could have, respectively?

- ○ $n-1$ and $n(n-1)/2$
- ○ $n-1$ and n^2
- ○ n and 2^n
- ○ n and n^n





Sparse vs. Dense Graphs

Let n = # of vertices, m = # of edges.

In most (but not all) applications, m is $\Omega(n)$ and $O(n^2)$.

- in a "sparse graph", m is $O(n)$ or close to it
- in a "dense graph", m is closer to $\Theta(n^2)$

The Adjacency Matrix

Represent G by a $n \times n$ 0-1 matrix A, where

$$A_{ij} = 1 \iff G \text{ has an } i\text{-}j \text{ edge} \qquad i - j$$

<u>Variants</u>

- A_{ij} = # of i-j edges (if parallel edges)
- A_{ij} = <u>weight</u> of i-j edge (if any)
- $A_{ij} = \begin{cases} +1 & \text{if } i \to j \\ -1 & \text{if } i \leftarrow j \end{cases}$

How much space does an adjacency matrix require, as a function of the number n of vertices and the number m of edges?

- $\theta(n)$
- $\theta(m)$
- $\theta(m+n)$
- $\theta(n^2)$

Adjacency Lists

<u>Ingredients</u>

- array (or list) of vertices
- array (or list) of edges
- each edge points to its endpoints
- each vertex points to edges incident on it

How much space does an adjacency list representation require, as a function of the number n of vertices and the number m of edges?

- $\theta(n)$
- $\theta(m)$
- $\theta(m + n)$
- $\theta(n^2)$

Adjacency Lists

<u>Ingredients</u>

- array (or list) of vertices
- array (or list) of edges
- each edge points to its endpoints
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one-to-one correspondence!

<u>Space</u>

$\Theta(n)$

$\Theta(m)$

$\Theta(m)$

$\Theta(m)$

$\Theta(m+n)$

[or $\Theta(\max\{m,n\})$]

<u>Question</u>: which is better?

<u>Answer</u>: depends on graph density and operations needed.

<u>This course</u>: focus adjacency lists.