Exact Algorithms for NP-Complete Problems

Algorithms: Design and Analysis, Part II

The Vertex Cover Problem

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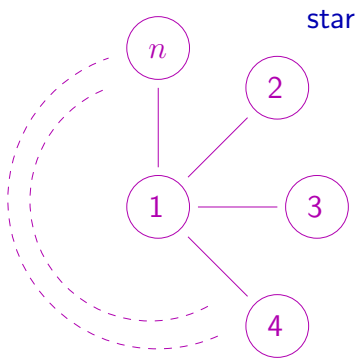
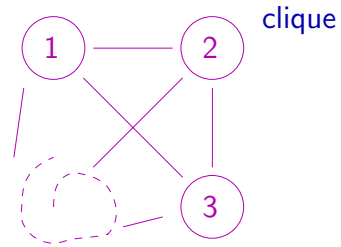
Input: An undirected graph $G = (V, E)$.

Goal: Compute a minimum-cardinality vertex cover – a subset $S \subseteq V$ that contains at least one endpoint of each edge of G.

Quiz

Question: What is the minimum size of a vertex cover of a star graph with n verities and a clique with n vertices respectively?





A) 1 and $n-1$

B) 1 and n

C) 2 and $n-1$

D) $n-1$ and n

Fact: In general, Vertex Cover is an NP-complete problem.

Strategies for NP-Complete Problems

(1) Identify computationally tractable special cases

- Trees [application of dynamic programming - try it!]
- Bipartite graphs [application of the maximum flow problem]
- When the optimal solution is "small" ($\approx \log n$ or less)

(2) Heuristics (e.g., via suitable greedy algorithms)

(3) Exponential time but better than brute-force search [coming up next]