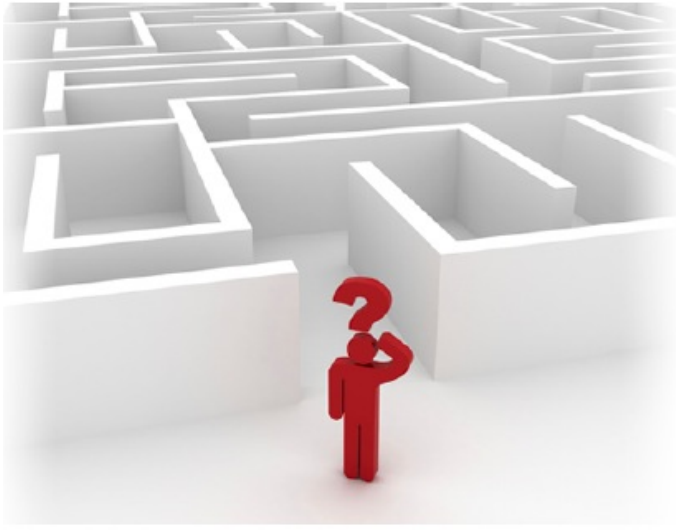


Minimum Spanning Trees

Algorithms: Design and Analysis, Part II

Correctness of Kruskal's Algorithm

Correctness of Kruskal (Part I)

Theorem: Kruskal's algorithm is correct.

Proof: Let $T^* =$ output of Kruskal's algorithm on input graph G.

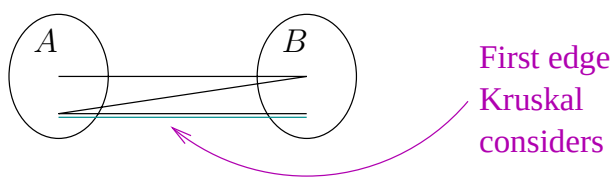
(1) Clearly T^* has no cycles.

(2) T^* is connected. Why?

(2a) By Empty Cut Lemma, only need to show that T^* crosses every cut.

(2b) Fix a cut (A, B). Since G connected at least one of its edges crosses (A, B).

Key point: Kruskal will include first edge crossing (A, B) that it sees [by Lonely Cut Corollary, cannot create a cycle]



Correctness of Kruskal (Part II)

(3) Every edge of T^* satisfied by the Cut Property. (Implies T^* is the MST)

Reason for (3): Consider iteration where edge (u, v) added to current set T. Since $T \cup \{(u, v)\}$ has no cycle, T has no $u - v$ path.

$\Rightarrow \exists$ empty cut (A, B) separating u and v. (As in proof of Empty Cut Lemma)

$\Rightarrow$ By (2b), no edges crossing (A, B) were previously considered by Kruskal's algorithm.

$\Rightarrow (u, v)$ is the first (+ hence the cheapest!) edge crossing (A, B).

$\Rightarrow (u, v)$ justified by the Cut Property. QED