Greedy Algorithms

A Scheduling Application: The Algorithm

Algorithms: Design and Analysis, Part II

Intuition for Algorithm

Recall: Want to $\min \sum_{j=1}^{n} w_j$.

Goal: Devise correct greedy algorithm.

Question:

1. With equal lengths, schedule larger or smaller-weight jobs earlier?
2. With equal weights, schedule shorter or longer jobs earlier?

A) Larger/shorter

C) Larger/longer

B) Smaller/shorter

D) Smaller/longer

Resolving Conflicting Advice

Question: What if $w_i > w_j$ but $l_i > l_j$?

Idea: Assign "scores" to jobs that are:

- inscreasing in weight
- decreasing in length

Guess (1): Order jobs by decreasing value of $w_j - l_j$.

Guess (2): Order w_j / l_j.

Breaking a Greedy Algorithm

To distinguish (1) & (2): Find example where the two algorithms produce different outputs. (At least one will be incorrect.)
Example:

$l_1 = 5, w_1 = 3$ (longer ratio)

$l_1 = 2, w_1 = 1$ (larger difference)

Question: What is the sum of weighted completion times of algorithms (1) & (2) respectively?

A) 22 and 23 C) 17 and 17

B) 23 and 22 D) 17 and 11

Alg#1: | #2 | #1 | $\rightarrow 1 \cdot 2 + 3 \cdot 7 = 23$

Alg#2: | #1 | #2 | $\rightarrow 3 \cdot 5 + 1 \cdot 7 = 22$

The Story So Far

So: Alg#1 not (always) correct.

Claim: Alg#2 (order by decreasing ratio w_j/l_j's) is always correct.
[not obvious! - proof coming up next]

Running time: $O(n \log n)$. [just need to sort]