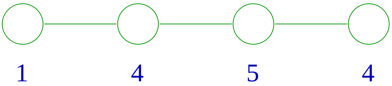
Dynamic Programming

Algorithms: Design and Analysis, Part II

Introduction, and WIS in Path Graphs

Problem Statement

Input: A path graph $G = (V, E)$ with nonnegative weights on vertices.



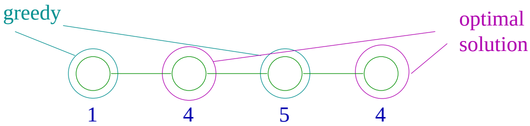
Desired output: Subset of nonadjacent vertices – an independent set – of maximum total weight.

Next: Iterate through our algorithm design principles.

Brute-force search: Requires exponential time.

A Greedy Approach

Greedy: Iteratively choose the max-weight vertex not adjacent to any previously chosen vertex.



Question: In the example, what is the value of the max-weight independent set, and that of the output of our greedy algorithm.

A) 14 and 10

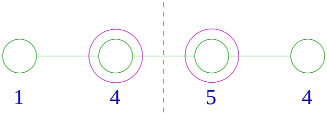
B) 8 and 6

C) 8 and 8

D) 9 and 8

A Divide & Conquer Approach

Idea: Recursively compute the max-weight IS of 1st half, ditto for 2nd half, then combine the solutions.



Problem: What if recursive sub-solutions conflict?
$\Rightarrow$ Not clear how to quickly fix.