All-Pairs Shortest Paths (APSP)

Algorithms: Design and Analysis, Part II

Problem Definition

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Input: Directed graph $G = (V, E)$ with edge costs c_e for each edge $e \in E$, [No distinguished source vertex.]

Goal: Either

(A) Compute the length of a shortest $u \rightarrow v$ path for all pairs of vertices $u, v \in V$

OR

(B) Correctly report that G contains a negative cycle.

Quiz

Question: How many invocations of a single-source shortest-path subroutine are needed to solve the all-pairs shortest path problem? [$n = \#$ of vertices]

A) 1

B) $n - 1$

C) n

D) n^2

Running time (nonnegative edge costs):

$$n \cdot \text{Dijkstra} = O(nm \log n) = \begin{array}{l} O(n^2 \log n) \text{ if } m = \Theta(n) \\ O(n^3 \log n) \text{ if } m = \Theta(n^2) \end{array}$$

Running time (general edge costs):

$$n \cdot \text{Bellman-Ford} = O(n^2 m) = \begin{array}{l} O(n^3) \text{ if } m = \Theta(n) \\ O(n^4) \text{ if } m = \Theta(n^2) \end{array}$$