Greedy Algorithms

A Scheduling Application: Correctness Proof Part I

Algorithms: Design and Analysis, Part II

Correctness Claim

Claim: Algorithm #2 (order jobs according to decreasing ratios w_j/l_j) is always correct.

Proof: By an Exchange Argument.

Plan: Fix arbitrary input of n jobs. Will proceed by contradiction.
Let σ = greedy schedule, σ^* = optimal schedule. (With σ^* better than σ.)
Will produce schedule even better than σ^*, contradicting purported optimality of σ^*.

Correctness Proof

Assume: All w_j/l_j's distinct.

Assume: [Just by renaming jobs] $w_1/l_1 > w_2/l_2 > \ldots > w_n/l_n$.

Thus: Greedy schedule σ is just $1, 2, 3, \ldots, n$.

Thus: If optimal schedule $\sigma^* \neq \sigma$, then there are consecutive jobs i, j with $i > j$.
[Only schedule where indices always go up is $1, 2, 3, \ldots, n$]

Correctness Proof (con'd)

So far:

1. $w_1/l_1 > w_2/l_2 > \ldots > w_n/l_n$
2. In optimal σ^*, $\exists$ consecutive jobs i, j with $i > j$.

Thought experiment: Suppose we exchange order of $i\&j$ in σ^* (leaving other jobs unchanged):