



Design and Analysis  
of Algorithms I


# Contraction Algorithm

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## The Algorithm

# The Minimum Cut Problem

- INPUT: An undirected graph  $G = (V, E)$ .

[ Parallel  edges allowed]

[See other video for representation of the input]

- GOAL: Compute a cut with fewest number of crossing edges. (a min cut)

# Random Contraction Algorithm

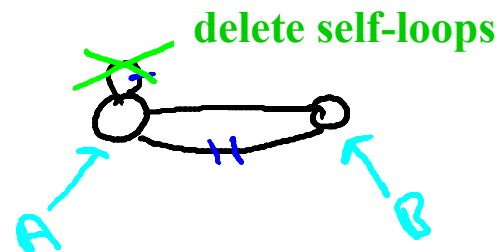
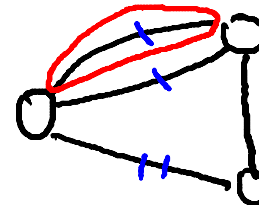
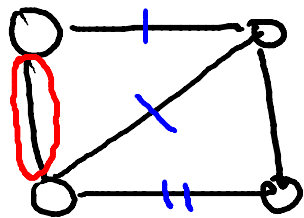
[ due to Karger, early 90s]

While there are more than 2 vertices:

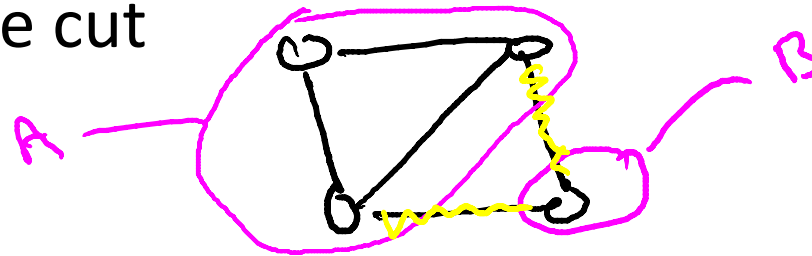
- pick a remaining edge  $(u,v)$  uniformly at random
- merge (or “contract” )  $u$  and  $v$  into a single vertex
- remove self-loops

return cut represented by final 2 vertices.

# Example



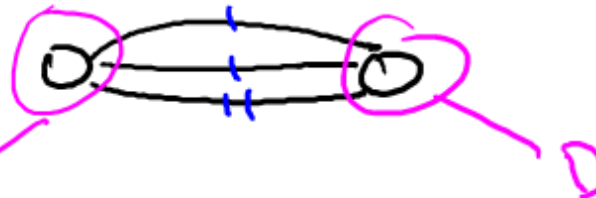
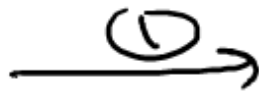
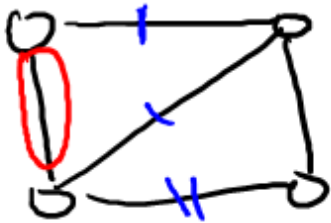
=> Corresponds to the cut



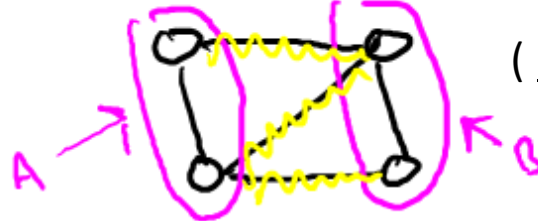
( a min cut! )

## Example (con'd)

**KEY  
QUESTION:**  
What is the  
probability of  
success?



➤ Corresponds to the cut



( not a min cut! )