



Design and Analysis
of Algorithms I

Contraction Algorithm

The Algorithm

The Minimum Cut Problem

- INPUT: An undirected graph $G = (V, E)$.

[Parallel  edges allowed]

[See other video for representation of the input]

- GOAL: Compute a cut with fewest number of crossing edges. (a min cut)

Random Contraction Algorithm

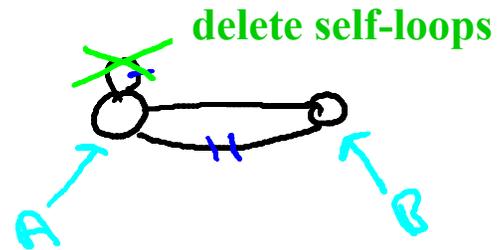
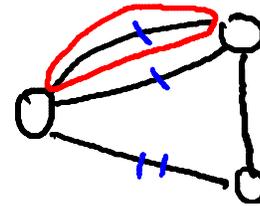
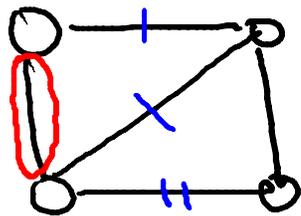
[due to Karger, early 90s]

While there are more than 2 vertices:

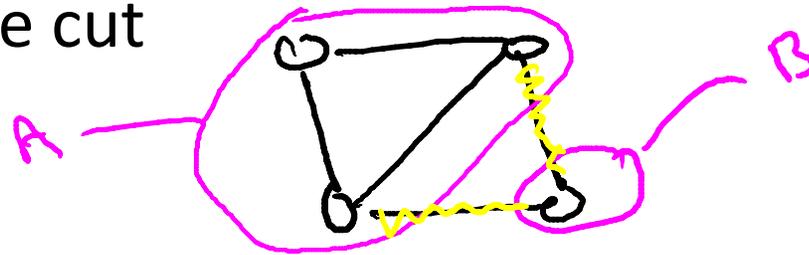
- pick a remaining edge (u,v) uniformly at random
- merge (or “contract”) u and v into a single vertex
- remove self-loops

return cut represented by final 2 vertices.

Example



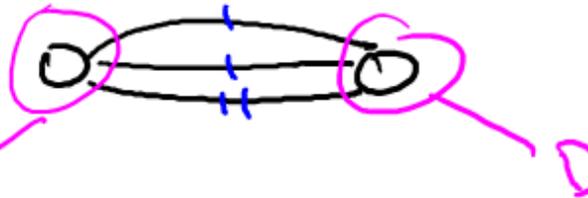
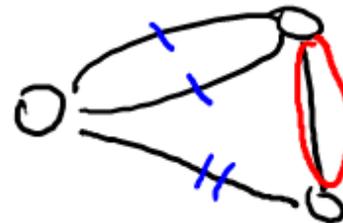
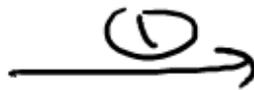
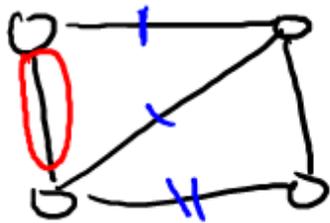
=> Corresponds to the cut



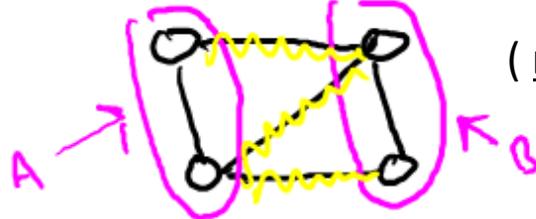
(a min cut!)

Example (con'd)

KEY QUESTION:
What is the probability of success?



➤ Corresponds to the cut



(not a min cut!)