



Design and Analysis  
of Algorithms I

# Master Method

## Motivation

# Integer Multiplication Revisited

Motivation : potentially useful algorithmic ideas often need mathematical analysis to evaluate

Recall : grade-school multiplication algorithm uses  $\theta(n^2)$  operation to multiply two n-digit numbers

# A Recursive Algorithm

## Recursive approach

Write  $x = 10^{n/2}a + b$        $y = 10^{n/2}c + d$   
[where a,b,c,d are n/2 – digit numbers]

So :

$$x \cdot y = 10^n ac + 10^{n/2}(ad + bc) + bd \quad (*)$$

Algorithm#1 : recursively compute ac,ad,bc,bd,  
then compute (\*) in the obvious way.

# A Recursive Algorithm

$T(n)$  = maximum number of operations this algorithm needs to multiply two  $n$ -digit numbers

Recurrence : express  $T(n)$  in terms of running time of recursive calls.

Base Case :  $T(1) \leq$  a constant.

For all  $n > 1$  :  $T(n) \leq 4T(n/2) + O(n)$

Work done  
here

Work done by recursive calls

# A Better Recursive Algorithm

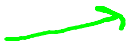
Algorithm #2 (Gauss): recursively compute  $ac^{(1)}$ ,  $bd^{(2)}$ ,  
 $(a+b)(c+d)^{(3)}$  [recall  $ad+bc = (3) - (1) - (2)$  ]

New Recurrence :

Base Case :  $T(1) \leq \text{a constant}$

Which recurrence best describes the running time of Gauss's algorithm for integer multiplication?

☐  $T(n) \leq 2T(n/2) + O(n^2)$

 ☒  $3T(n/2) + O(n)$

☐  $4T(n/2) + O(n)$

☐  $4T(n/2) + O(n^2)$

# A Better Recursive Algorithm

Algorithm #2 (Gauss): recursively compute  $ac^{(1)}$ ,  $bd^{(2)}$ ,  
 $(a+b)(c+d)^{(3)}$  [recall  $ad+bc = (3) - (1) - (2)$  ]

New Recurrence :

Base Case :  $T(1) \leq$  a constant

For all  $n > 1$  :  $T(n) \leq 3T(n/2) + O(n)$

Work done  
here



Work done by recursive calls

