Bonus Lecture #5: SNARKs Under the Hood

COMS 4995-001: The Science of Blockchains

URL: https://timroughgarden.org/s25/

Tim Roughgarden

Goals for Bonus Lecture #5

1. Review of NP and SNARKs.
   - SNARK = succinct, noninteractive argument of knowledge
   - short (<< witness length) & easy-to-verify proofs of an NP statement
2. General probabilistic verification and the PCP Theorem.
   - every NP problem can be probabilistically verified
3. PCP Theorem ➔ SNARKs.
   - can derive SNARKS from one of the deepest results in theory CS
4. Bird's-eye view of modern SNARK constrictions.
   - front ends, back ends, polynomial commitments, polynomial IOPs

Review: Witnesses and NP Statements

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Idea of a SNARK: proof that x a “yes” instance, with proof length << witness length and verification time << time to compute C(x,w).

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- given w with C(x,w)=1, easy to generate π s.t. V(x, π)="yes"
  - "prover time," ideally O(RT of C) with non-astronomical hidden constant
- if x a "no" instance, computationally infeasible to find π s.t. V(x, π)="yes"
  - i.e., practically impossible to convince verifier of a false statement

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Key ingredients: (cf., matrix multiplication)

- probabilistic verification (catches false claims e.g. 50% of the time)
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Question: how to probabilistically verify arbitrary computations?

Amazing fact: *every* NP problem can be probabilistically verified.

Recall: The 3-SAT Problem

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Problem: 3-SAT

Input: A list of Boolean decision variables $x_1, x_2, \ldots, x_n$; and a list of constraints, each a disjunction of at most three literals.

Output: A truth assignment to $x_1, x_2, \ldots, x_n$ that satisfies every constraint, or a correct declaration that no such truth assignment exists.

For example, there's no way to satisfy all eight of the constraints

$$\begin{array}{cccc} x_1 \vee x_2 \vee x_3 & x_1 \vee \neg x_2 \vee x_3 & \neg x_1 \vee \neg x_2 \vee x_3 & x_1 \vee \neg x_2 \vee \neg x_3 \\ \neg x_1 \vee x_2 \vee x_3 & x_1 \vee x_2 \vee \neg x_3 & \neg x_1 \vee x_2 \vee \neg x_3 & \neg x_1 \vee \neg x_2 \vee \neg x_3, \end{array}$$

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Cook-Levin Theorem: 3-SAT is NP-complete.

- i.e., *every* NP problem can be encoded as 3-SAT.

Proof of the Cook-Levin Theorem

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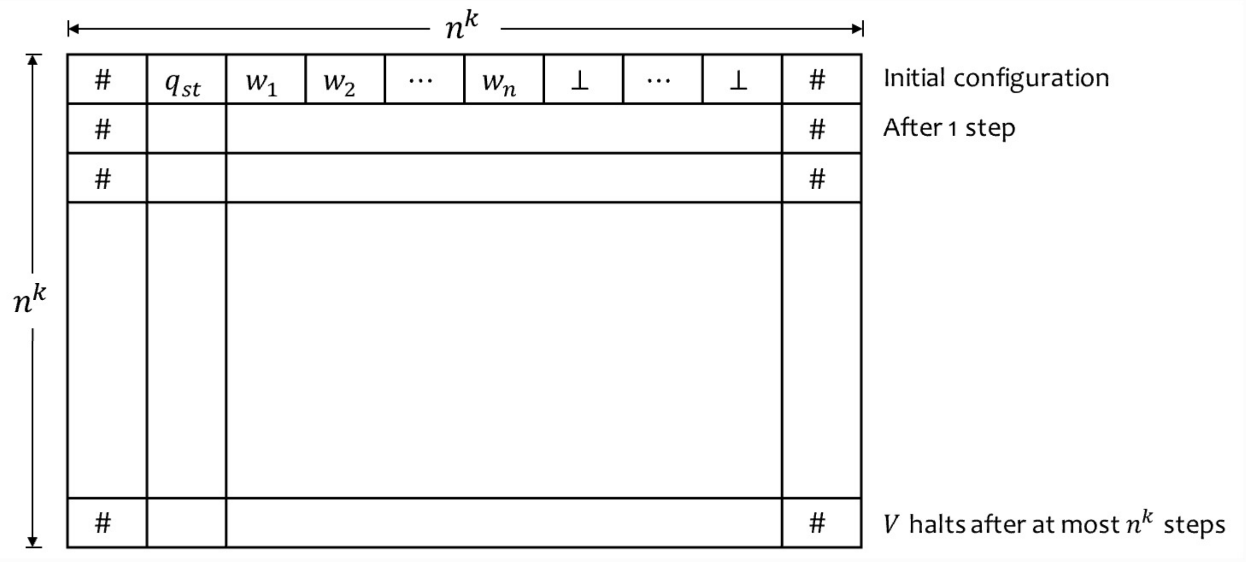
- proof idea: decision variables = state of memory at each time step of computation; constraints = computation proceeds according to C



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Upshot: to probabilistically verify every NP problem, enough to probabilistically verify 3-SAT.

– verify an arbitrary NP problem by first converting it to 3-SAT

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Because SAT is NP-complete: *every* NP problem L can be likewise probabilistically verified. [Convert L to 3-SAT, use PCP theorem.]

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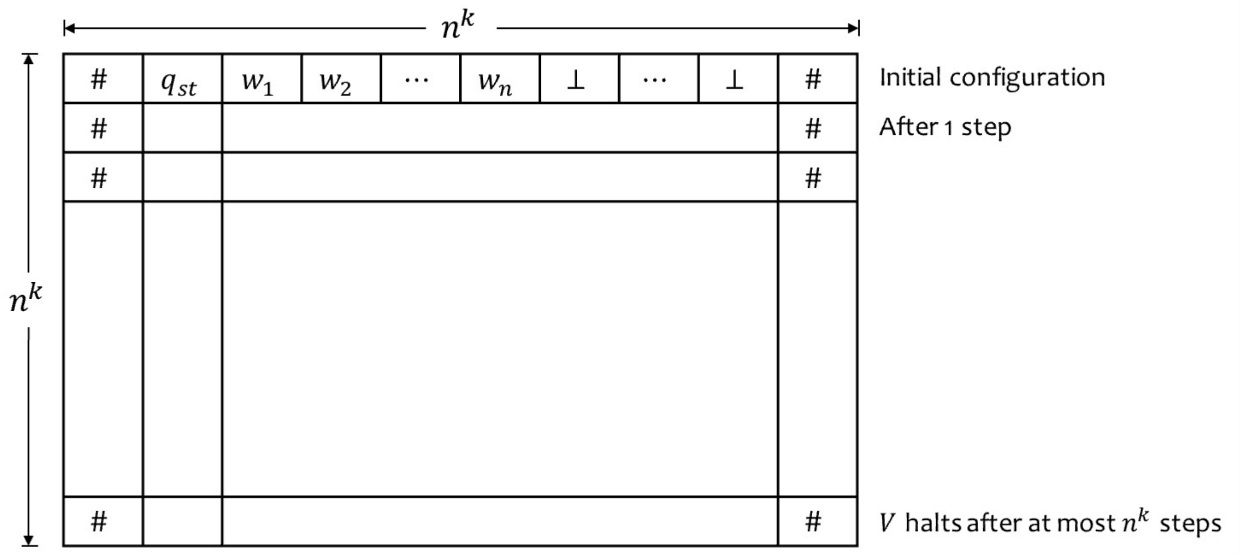

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Good news: (i) same correctness guarantees as matrix multiplication (ii) fast verification [O(t) evaluations of h, random accesses to y]

Bad news: proof y is not succinct (at least as large as witness length).

– y is essentially a redundantly encoded satisfying truth assignment

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- prover forms a Merkle tree T with leaves = bits of y, posts root r of T

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  - O(t) Merkle proofs in all

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Result: correctness same as before (assuming no hash fn collisions), proof size now O(log RT(C)). [assuming t = O(1)]

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SNARK for an arbitrary NP problem:

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- prover posts r, allegedly the root of a Merkle whose leaves encode a PCP proof for a satisfying assignment of φ_x (i.e., φ w/bits of x fixed accordingly)
- prover posts t · q Merkle proofs (q = # of queries made by PCP verifier)
- verification: for i=1,2,…,t:
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Conclusion: SNARKs exist for arbitrary NP problems!

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Conclusion: SNARKs exist for arbitrary NP problems!

- not immediately practical (too much work to generate PCP proof y)
- but the conceptual basis for modern SNARK constructions

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Practical SNARKs: require careful joint optimization of the front end, the polynomial commitment scheme, and the polynomial IOP.