Quantifying Inefficiency in Games and Mechanisms

Tim Roughgarden (Stanford)

Talk Themes

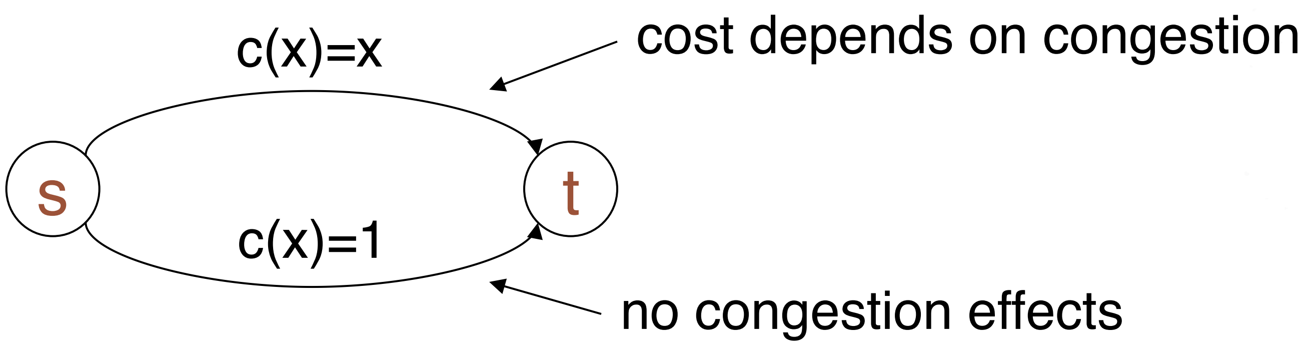
- many economic concepts directly relevant for reasoning about applications in computer science
  - Shapley value, correlated equilibria, etc.

- tools from computer science can yield new insights into basic economic models
  - transportation networks, cost-sharing, etc.
  - using approximation to reason about efficiency loss

- shared concern: theory to guide design
  - traditional approaches: axiomatic, optimization
  - here: minimize worst-case efficiency loss

Pigou's Example

Example: one unit of traffic wants to go from s to t

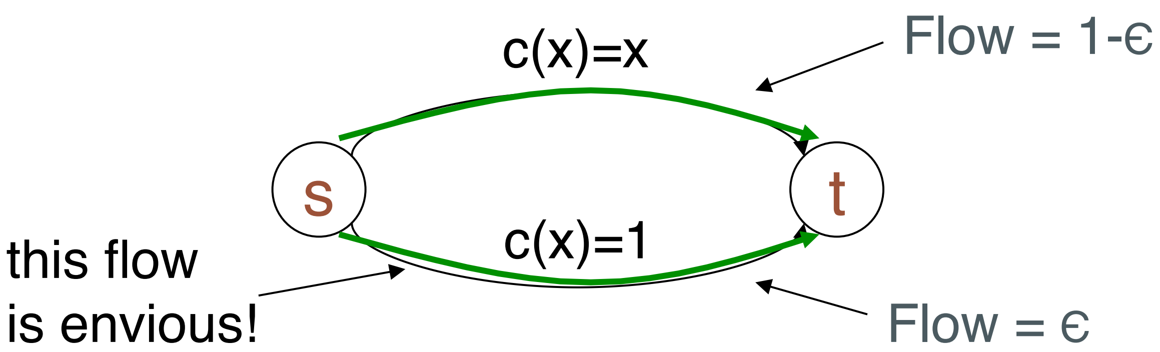


Question: what will selfish drivers do?

- assume everyone wants smallest-possible cost
- [Pigou 1920]

Equilibrium in Pigou's Example

Claim: all traffic will take the top link.

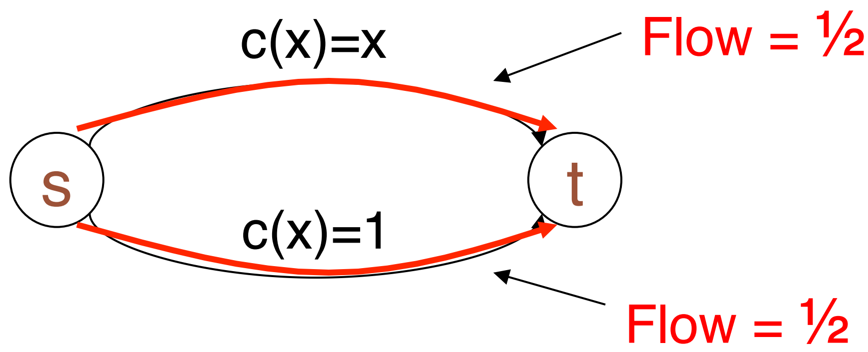


Reason:

- є > 0 => traffic on bottom is envious
- є = 0 => equilibrium
  - all traffic incurs one unit of cost

Can We Do Better?

Consider instead: traffic split equally

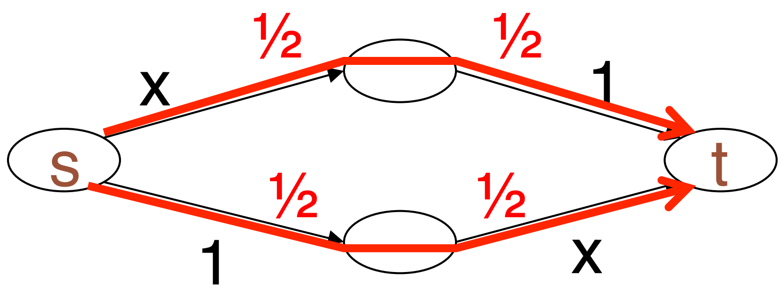


Improvement:

- half of traffic has cost 1 (same as before)
- half of traffic has cost ½ (much improved!)
- "price of anarchy" [Kousoupias/Papadimitriou 99] = 4/3

Braess's Paradox

Initial Network:



Cost = 1.5

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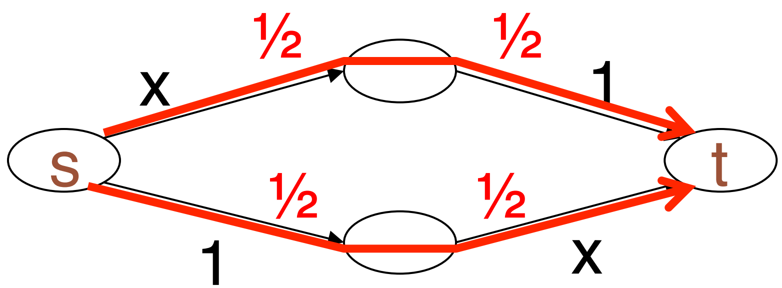

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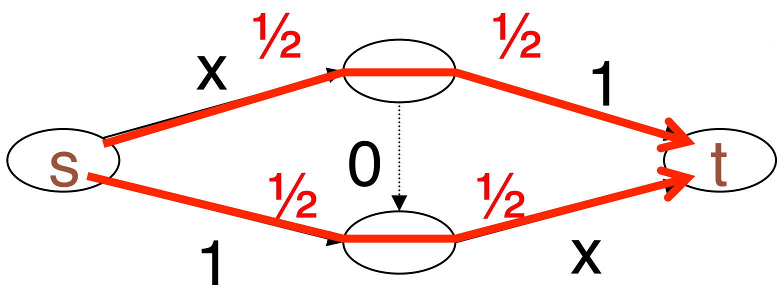
Augmented Network:



Now what?

Braess's Paradox

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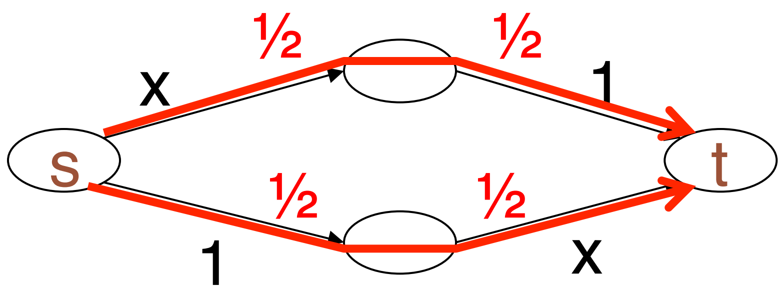

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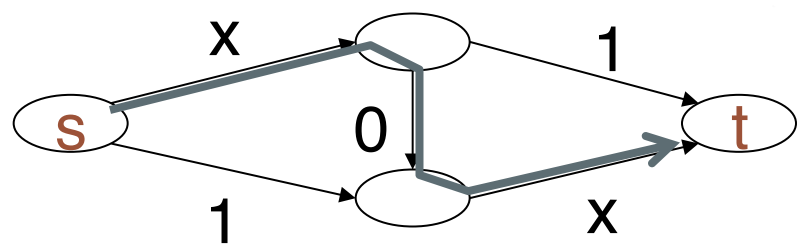
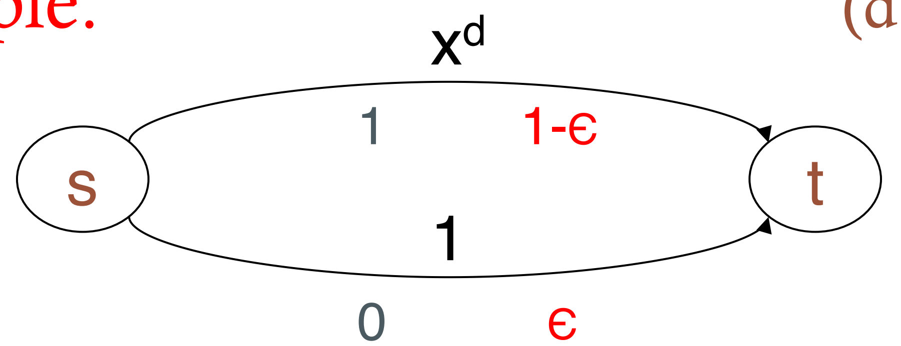


Cost = 2

Price of anarchy = 4/3 in augmented network (again!)

A Nonlinear Pigou Network

Bad Example: (d large)



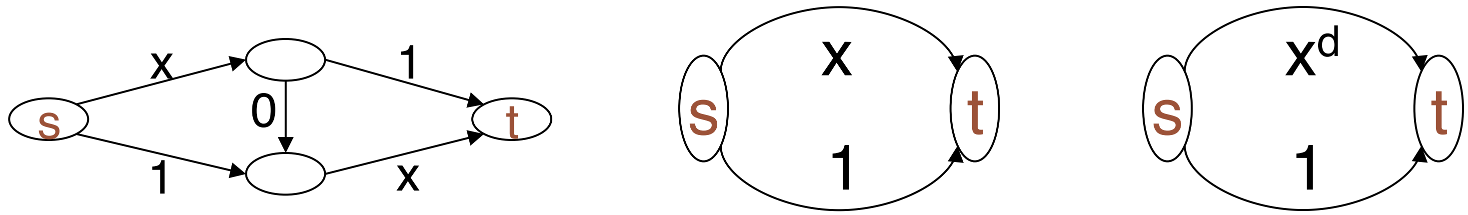
equilibrium has cost 1, min cost -> 0

=> price of anarchy unbounded as d -> infinity

Goal: weakest-possible conditions under which the price of anarchy is small.

When Is the Price of Anarchy Bounded?

Examples so far:



Hope: imposing additional structure on the cost functions helps

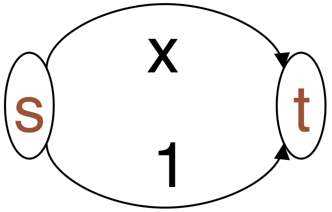
- worry: bad things happen in larger networks

Affine Cost Functions

Defn: affine cost function is of form $c_e(x)=a_e x+b_e$

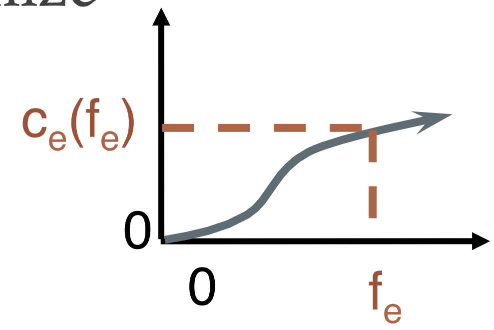
Theorem: [Roughgarden/Tardos 00] for every network with affine cost functions:

cost of eq flow $\leq$ 4/3 $\times$ cost of opt flow



The Potential Function

Easy fact: [BMW 56] Nash flows minimize "potential function" $\sum_e \int_0^{f_e} c_e(x)\,dx$ (over all flows).



Proof: FOC + convexity.

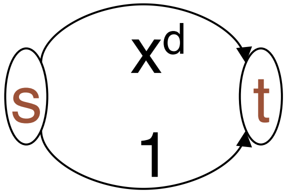
Corollary: for affine cost functions:

- cost, potential functions differ by factor of ≤ 2
- gives upper bound of 2 on price on anarchy

$$C(f) \leq 2\times PF(f) \leq 2\times PF(f^*) \leq 2\times C(f^*)$$

General Cost Functions

Theorem: [Roughgarden 02], [Correa/Schulz/Stier Moses 03] fix any set of cost functions. Then, a Pigou-like example --- 2 nodes, 2 links, 1 link w/ a constant cost function --- achieves the worst P.O.A.



tight example

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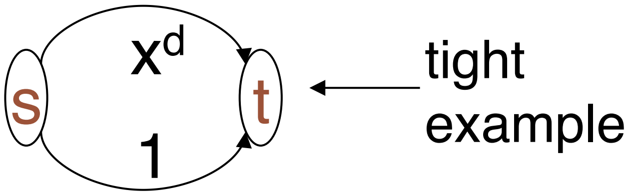
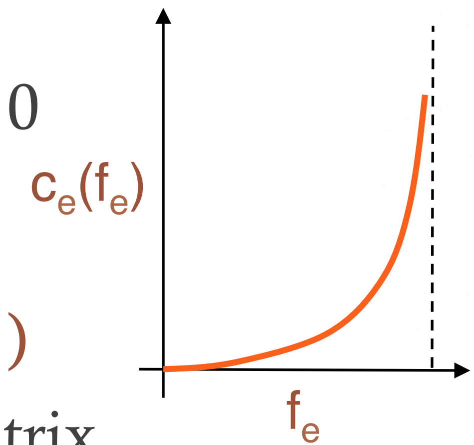


Take-away: worst-case inefficiency governed by cost function nonlinearity, not network structure.

Benefit of Overprovisioning

M/M/1 Cost Functions: $c(x) = 1/(u_e\text{-}x)$



Suppose: network is overprovisioned by $\beta > 0$ (i.e., β fraction of each edge unused).

Then: Price of anarchy is at most $\frac{1}{2}(1+1/\sqrt{\beta})$

- arbitrary network size/topology, traffic matrix

Moral: Even modest (10%) over-provisioning sufficient for near-optimal routing.

Outline

1. The price of anarchy in routing games.

2. *Learnable equilibria: robust POA bounds.*
    1. *Connections to learning in games.*
    2. *POA bounds: the next generation.*

3. Cost-sharing mechanisms: worst-case inefficiency.

4. Optimal cost-sharing in routing games.

5. Simple auctions for complex settings.

POA Bounds Without Convergence

Meaning of a POA bound: *if* the game is at an equilibrium, *then* outcome is near-optimal.

Problem: what if can't reach an equilibrium?

- non-existence (pure Nash equilibria)
- intractability (mixed Nash equilibria) [Daskalakis/Goldberg/Papadimitriou 06], [Chen/Deng/Teng 06], [Etessami/Yannakakis 07]
- hard to learn Nash equilibria [Hart/Mas-Colell 03], …

Worry: fail to converge, so POA bound doesn't apply.

Learnable Equilibria

Fact: simple strategies converge quickly to more permissive equilibrium sets.

- correlated equilibria: [Foster/Vohra 97], [Fudenberg/Levine 99], [Hart/Mas-Colell 00], …
- coarse/weak correlated equilibria (of [Moulin/Vial 78]): [Hannan 57], [Littlestone/Warmuth 94], …

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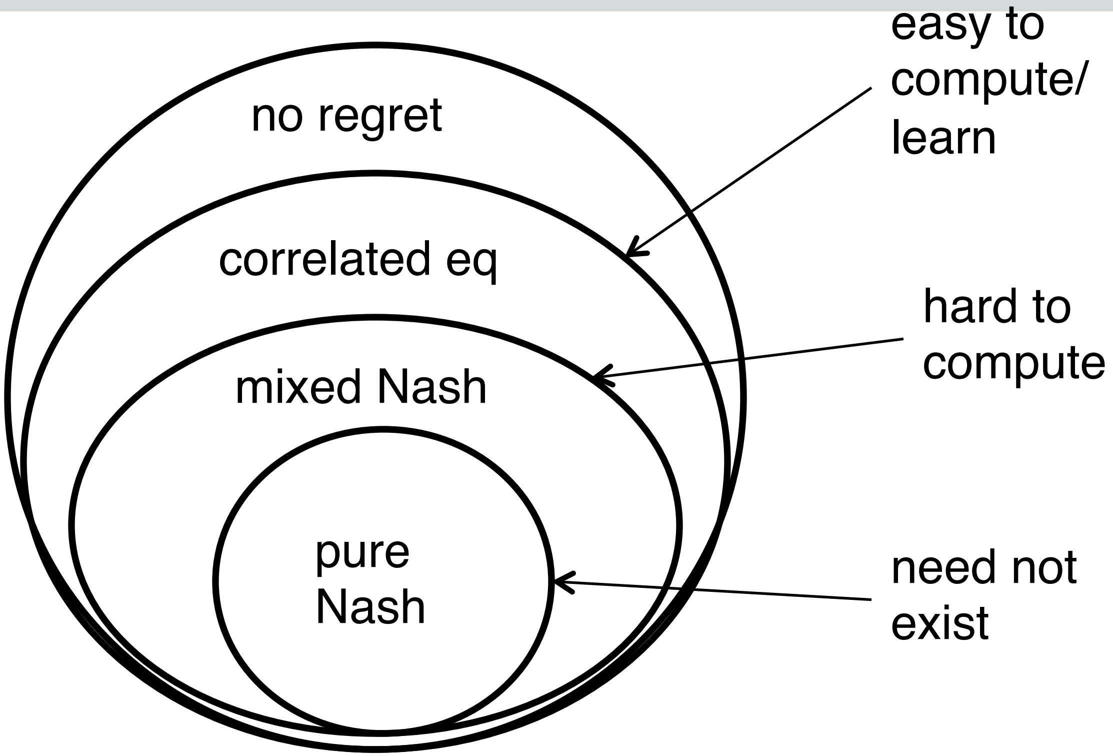
Question: are there good "robust" POA bounds, which hold more generally for such "easily learned" equilibria?

[Mirrokni/Vetta 04], [Goemans/Mirrokni/Vetta 05], [Awerbuch/Azar/Epstein/Mirrokni/Skopalik 08], [Christodoulou/Koutsoupias 05], [Blum/Even-Dar/Ligett 06], [Blum/Hajiaghayi/Ligett/Roth 08]

A Hierarchy of Equilibria



Recall: POA determined by *worst* equilibrium (only increases with the equilibrium set).

POA Bounds Without Convergence

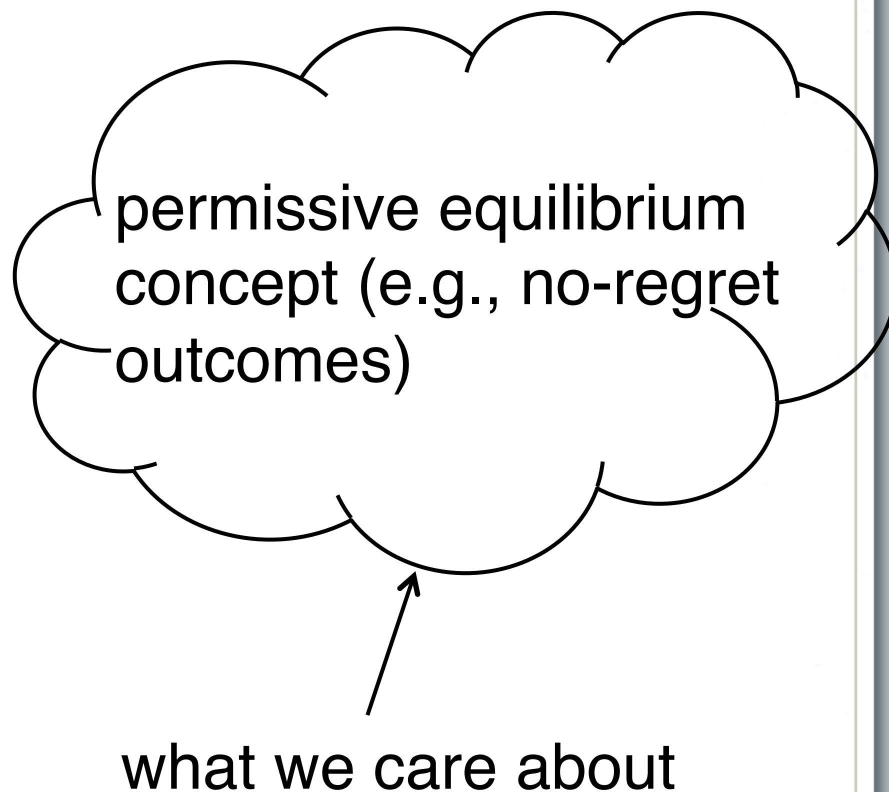
Theorem: [Roughgarden 2009] most known POA bounds hold even for all coarse correlated equilibria.

Part I: [extension theorem] every POA bound proved for pure Nash equilibria *in a prescribed way* extends automatically, with no quantitative loss, to all no-regret outcomes.

- eludes non-existence/intractability critiques.

Part II: most known POA bounds were proved in this way (so extension theorem applies).

Extension Theorems



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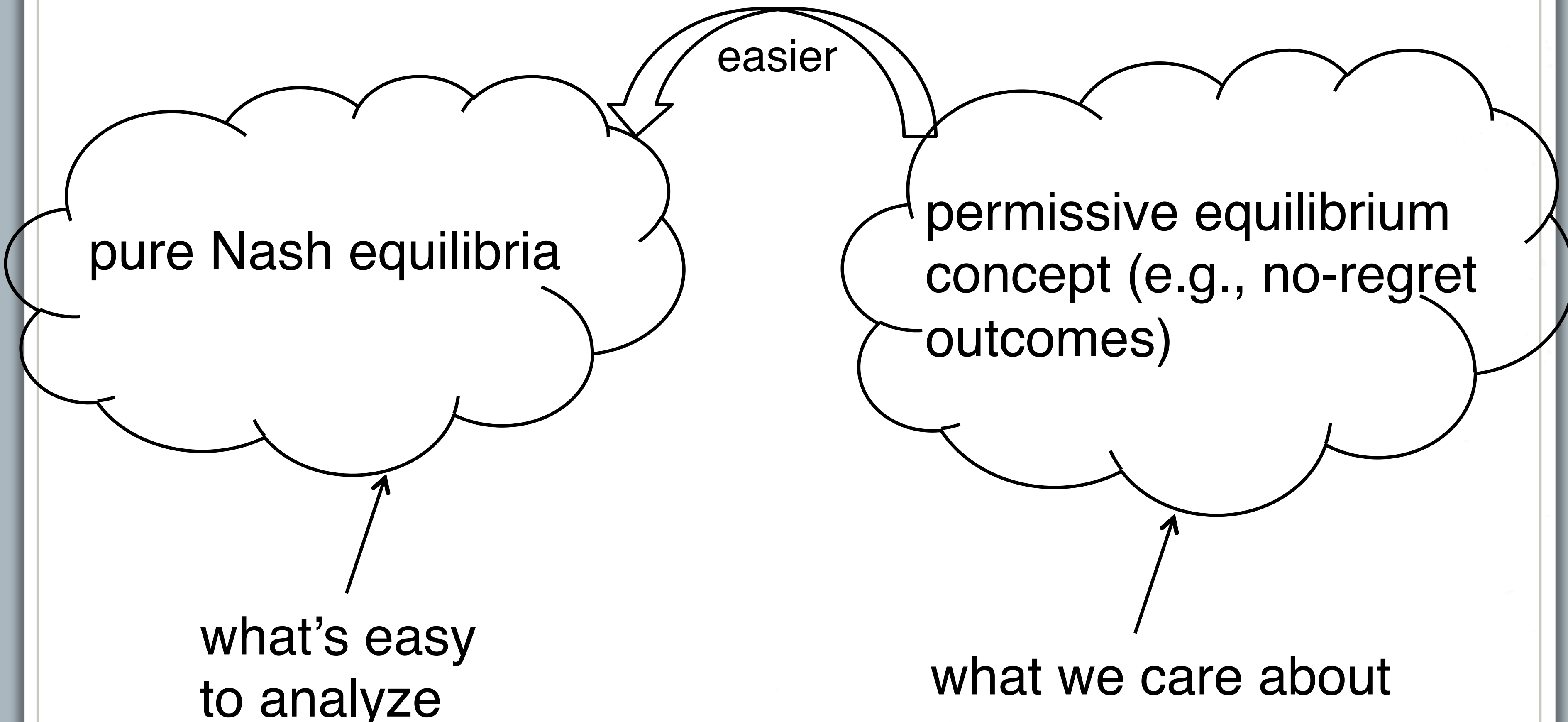

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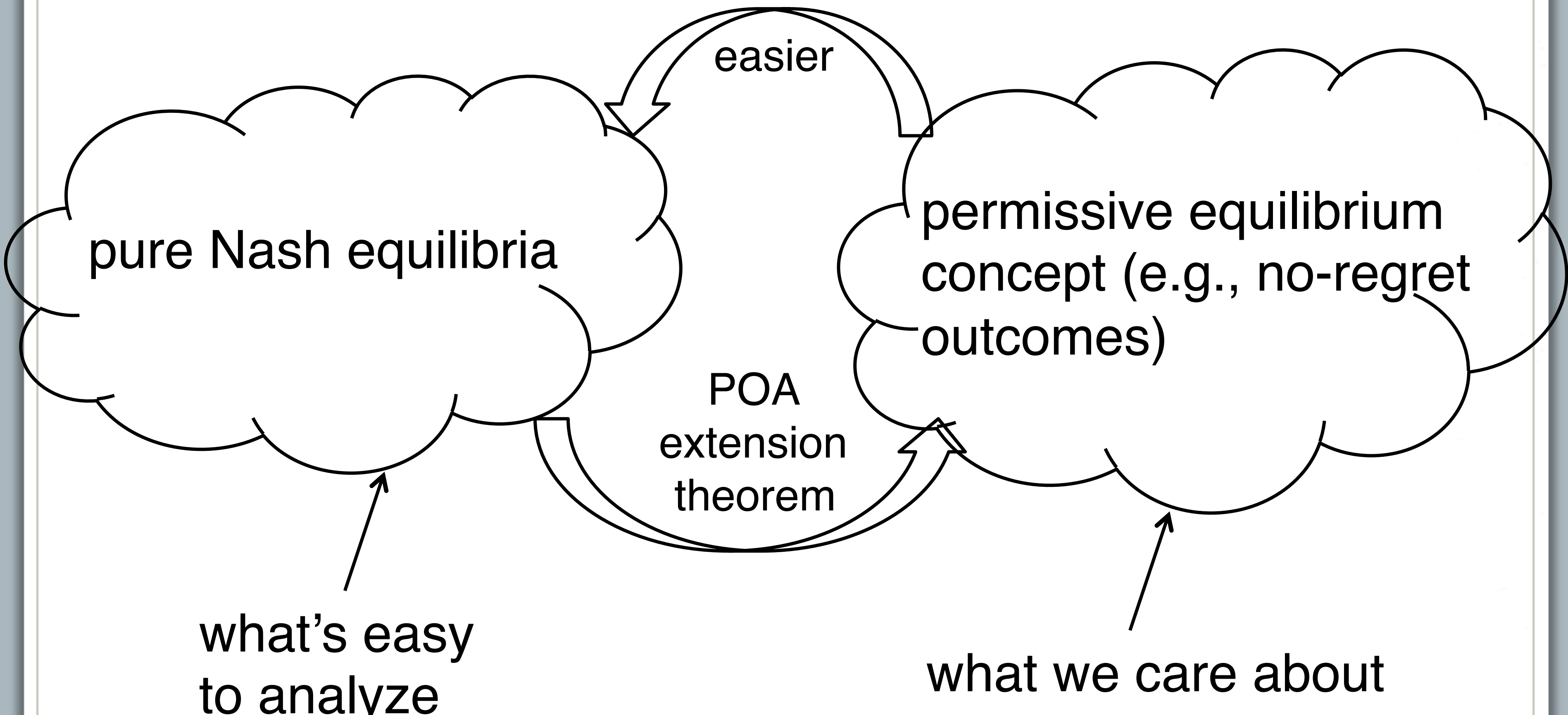

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1. The price of anarchy in routing games.
2. Learnable equilibria: robust POA bounds.
3. *Cost-sharing mechanisms: worst-case inefficiency.*
   1. *Inefficiency in mechanism design.*
   2. *Designing to minimize worst-case efficiency loss.*
4. Optimal cost-sharing in routing games.
5. Simple auctions for complex settings.

Public Excludable Good

E.g., [Deb/Razzolini 99], [Moulin 99]

- player i has valuation v_i for winning
- welfare of S = v(S) - C(S) [where $v(S) = \Sigma_i v_i$]
- $C(ø) = 0$, $C(S) = 1$ if $S \neq ø$

Constraints: want a dominant strategy IC + IR, budget-balanced mechanism, no positive transfers.

- [Green/Laffont 79] efficiency loss inevitable

Design goal: mechanism with minimum worst-case loss.

Equal-Share Mechanism

The Mechanism: (which is BB+DSIC, even GSP)

- collects bids (b_i for each i)
- initialize S = all players
- if $b_i \geq 1/|S|$ for all i in S, halt.
- else drop a player i with $b_i < 1/|S|$ and iterate

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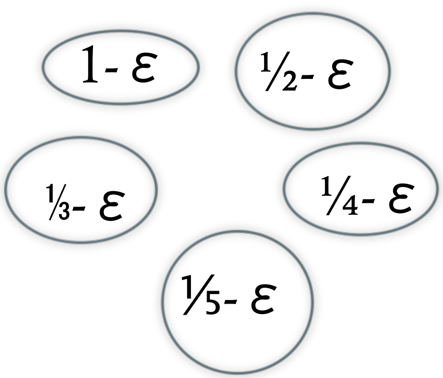
Efficiency loss:

- set $v_i = (1/i) - \varepsilon$ for i=1,2,…,k
- max welfare ≈ ln k – 1
- mechanism welfare = 0



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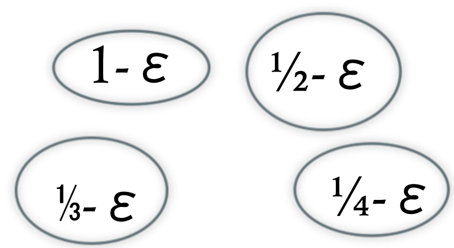

Worst-Case Efficiency Loss

Theorems: [Moulin/Shenker 01, Roughgarden/Sundararajan 06]

- ≈ (ln k – 1) is worst-case welfare loss.
- analogous results for general submodular ("diminishing marginal costs") functions
  - ex: C(S) = concave function of player weights in S
- analog of equal-split: cost shares = Shapley values
- worst-case welfare loss = value of potential function defined in [Hart/Mas-Colell 90]
  - corresponds also to a simple worst-case example
  - never worse than for a public excludable good
  - less severe as C gets "more linear"

Can We Do Better?

Cost-sharing method: assigns *cost share* χ (i,S) to each player i in S (for every set S). (e.g., Shapley value)

- constraints: budget-balance, "cross-monotonicity"

Question: which cost-sharing method minimizes the worst-case efficiency loss (over all valuation profiles)?

Theorem: [Moulin/Shenker 01, Roughgarden/Sundararajan 06] the Shapley value is optimal (any submodular cost fn).

Extensions: [Dobzinski et al 08], [Juarez 13], ...

Cost Sharing at Google

Sundararajan

Motivation: *attribution problems.*

- marketer or advertiser compares Q1 vs Q2 revenue
- suppose several variables changed in Q2
  - better targeting
  - better matching algorithms
  - stronger economy
- what percentage of change to attribute to each variable?
  - essentially a budget-balanced cost-sharing problem!

Theorem: [Sun/Sundararajan 11] axiomatic justification of Aumann-Shapley value for multi-linear revenue fns.

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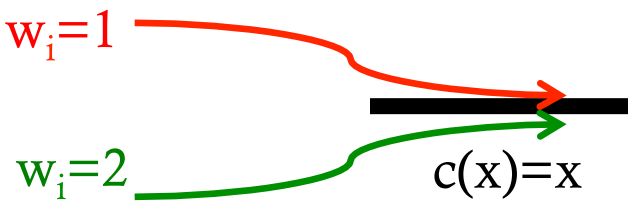
4. *Optimal cost sharing in routing games.*
   1. *Designing to minimize the POA.*
   2. *More magical properties of the Shapley value.*
5. Simple auctions for complex settings.

Routing Games, Revisited

Weighted routing games: (w/finite number of players)

- player i has origin s_i, destination t_i, chooses an s_i-t_i path on which to route w_i units of traffic
- cost on edge e: $c_e(f_e)f_e$, where f_e = total weight using e



Design question: how to share joint cost among users?

Traditional answer: proportional to players' weights.

Can We Do Better?

Proportional Cost Sharing:

- corresponds to a FIFO queueing policy [Shenker 95]
- pure Nash equilibria need not exist [Rosenthal 73]
- worst-case POA well understood; modest if cost funtions "close to linear" [Awerbuch/Azar/Epstein 05], [Christodoulou/Koutsoupias 05], [Aland et al 06]

Design Questions:

- can different cost shares restore pure Nash equilibria?
- can different cost shares reduce the worst-case POA?

Restoring Pure Equilibria

Theorem: [Kollias/Roughgarden 11] sharing costs using a weighted Shapley value ([Shapley 53], [Kalai/Samet 87]) induces a potential game ([Monderer/Shapley 96]).

- also, worst-case POA of unweighted Shapley value only slightly bigger than with proportional cost-sharing

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Theorem: [Gopalakrishnan/Marden/Wierman 13] these are the *only* cost-sharing methods guaranteed to induce a pure Nash equilibrium in every weighted routing game!

- a potential is *necessary* for guarnateed existence of PNE
- extension of [Chen/Roughgarden/Valiant 08]

Minimizing the POA

Theorem: [Gkatzelis/Kollias/Roughgarden 14] among all budget-balanced cost-sharing methods that guarantee existence of pure Nash equilibria, the Shapley value minimizes the worst-case POA in weighted routing games.

- slightly worse POA than with proportional cost-sharing

Theorem: [Gkatzelis/Kollias/Roughgarden 14] among all budget-balanced cost-sharing methods, proportional cost shares minimize the worst-case POA in weighted routing games.

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Motivating Question

Question: When can simple auctions perform well in complex settings?

Example: welfare maximization with multiple non-identical goods (combinatorial auctions).

- theoretically optimal: VCG mechanism
- simple: selling items separately
  - when is equilibrium welfare close to optimal?
- example interpretation: is package bidding essential to good combinatorial auction designs?

The POA of Simple Auctions

[Christodoulou/Kovacs/Schapira 08], [Lucier/Borodin 10],

[Paes Leme/Tardos 10], [Bhawalkar/Roughgarden 11],

[Hassidim/Kaplan/Mansour/Nisan 11], [Lucier/Paes Leme 11],

[Caragiannis/Kaklamanis/Kanellopoulos/Kyropoulou 11],

[Lucier/Singer/Syrgkanis/Tardos 11], [Markakis/Telelis 12],

[Paes Leme/Syrgkanis/Tardos 12], [Bhawalkar/Roughgarden 12], [Feldman/Fu/Gravin/Lucier 13], [Syrgkanis/Tardos 13],

[de Keijzer/Markakis/Schaefer/Telelis 13],

[Duetting/Henzinger/Starnberger 13],

[Babaioff/Lucier/Nisan/Paes Leme 13],

[Devanur/Morgenstern/Syrgkanis 13], …

The High-Order Bits

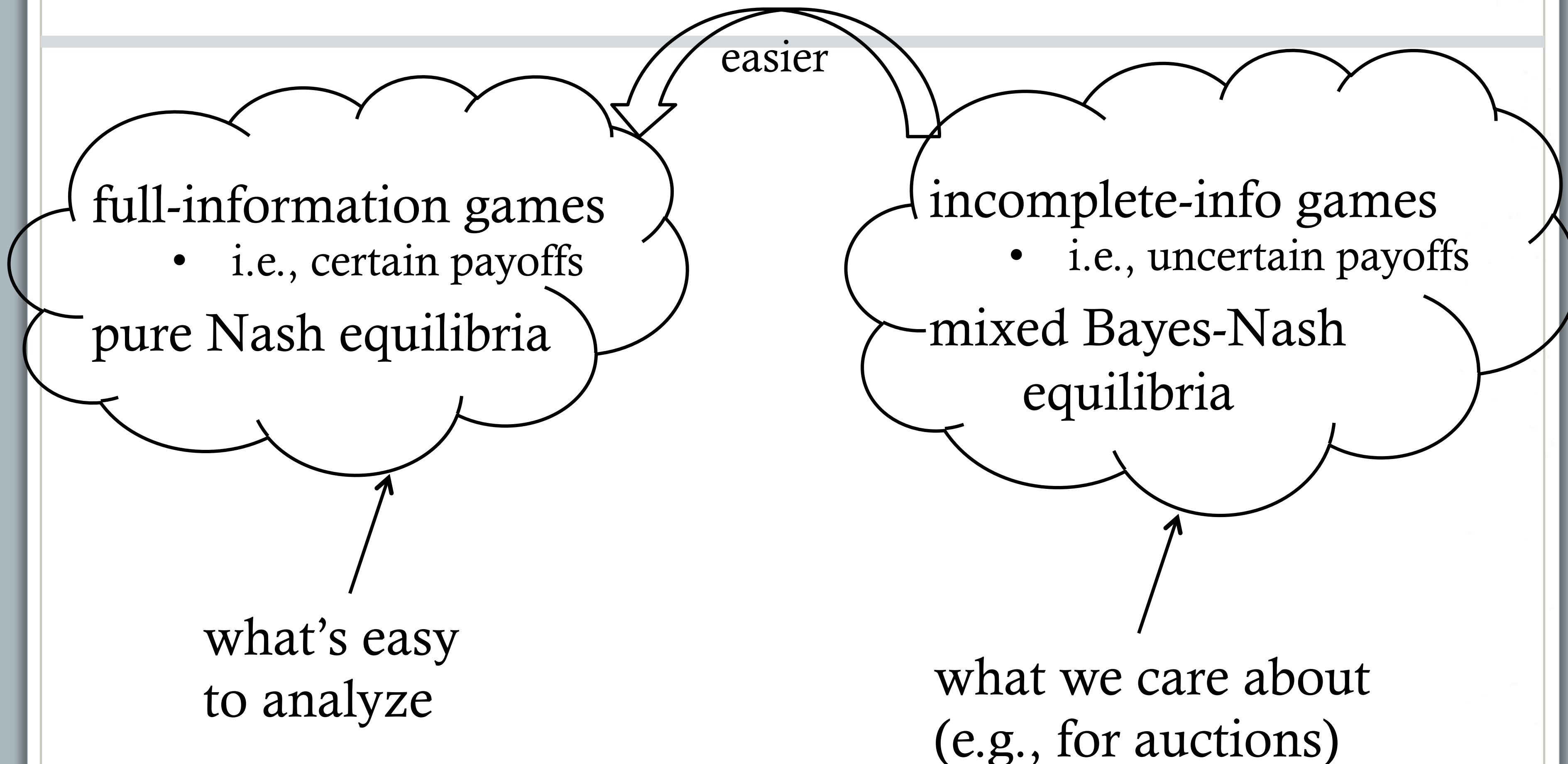
incomplete-info games

- i.e., uncertain payoffs

mixed Bayes-Nash equilibria

what we care about (e.g., for auctions)

The High-Order Bits



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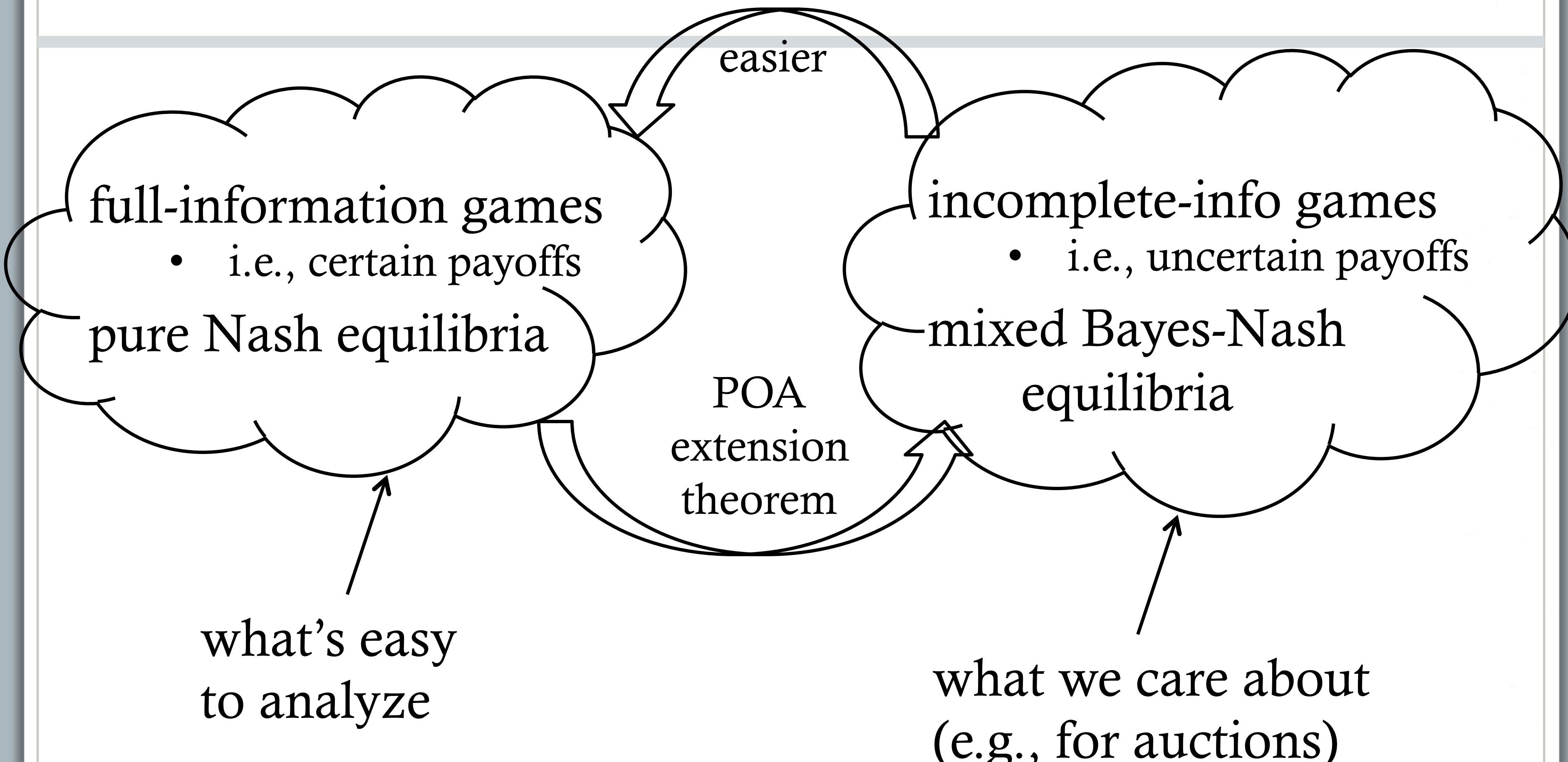

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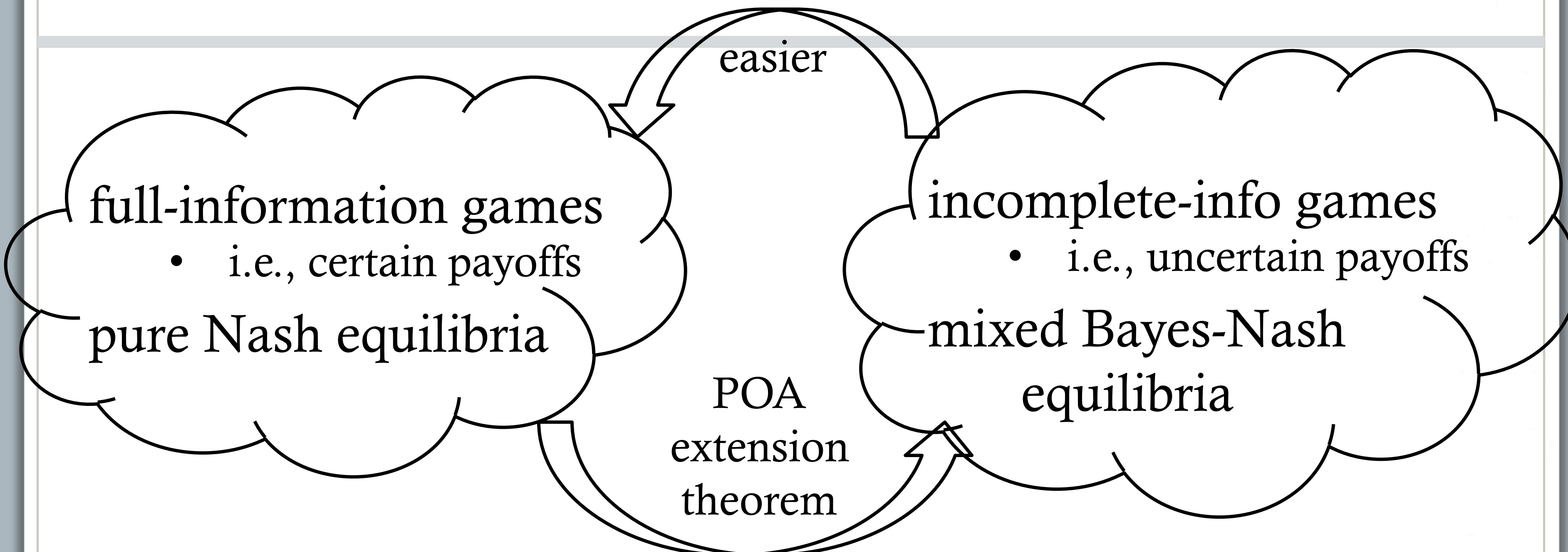

- extension theorems for Bayes-Nash equilibria: [Roughgarden 12], [Syrgkanis/Tardos 13]

Concluding Remarks

- reasoning about inefficiency through approximation gives new insights into fundamental economic models
  - try applying these ideas to your favorite model!
- good bounds for many games of interest, even for easy-to-learn equilibria
- crisp advice for designing mechanisms and systems
  - overprovisioning communication networks
  - the many magical properties of the Shapley value