CS364A: Exercise Set #5

Due by the beginning of class on Wednesday, October 30, 2013

Instructions:

(1) Turn in your solutions to all of the following exercises directly to one of the TAs (Kostas or Okke). Please type your solutions if possible and feel free to use the LaTeX template provided on the course home page. Email your solutions to `cs364a-aut1314-submissions@cs.stanford.edu`. If you prefer to hand-write your solutions, you can give it to one of the TAs in person at the start of the lecture.

(2) Your solutions will be graded on a "check/minus/zero" system, with "check" meaning satisfactory and "minus" meaning needs improvement.

(3) Solve these exercises and write up your solutions on your own. You may, however, discuss the exercises verbally at a high level with other students. You may also consult any books, papers, and Internet resources that you wish. And of course, you are encouraged to contact the course staff (via Piazza or office hours) to clarify the questions and the course material.

(4) No late assignments will be accepted.

Lecture 9 Exercises

Exercise 35

Consider single-item auctions with known bidder budgets. Give a DSIC auction (possibly randomized) that is guaranteed to achieve (in expectation) at least a $\frac{1}{n}$ fraction of the maximum-possible surplus $\sum_i v_i x_i$.

Exercise 36

Continuing the previous exercise, prove that for every DSIC auction (possibly randomized), there is a valuation profile on which its expected surplus is $O(1/n)$ times the maximum possible. For partial credit, prove this for deterministic DSIC auctions only.

Exercise 37

Consider a 0-1 single-parameter environment (as in Exercise 15) in which each bidder i has a publicly known budget B_i. Consider the allocation rule $\mathbf{x}$ that, given bids $\mathbf{b}$, chooses the feasible outcome that maximizes the "truncated welfare" $\sum_{i=1}^n \min\{b_i x_i, B_i\}$. Argue that this rule is monotone (assuming consistent tie-breaking), and that the corresponding DSIC mechanism (given by Myerson's Lemma) never charges a bidder more than its budget.

Exercise 38

Continuing the previous exercise, give a plausible argument why the mechanism in Exercise 37 might be a reasonable approach to single-item auctions when bidders have publicly known budgets.

Exercise 39

Continuing the previous exercise, consider instead the setting of the clinching auction — a possibly large number m of identical goods, and each bidder i has a private valuation v_i per good and a publicly known budget B_i. Give an argument why the mechanism in Exercise 37 might not be a reasonable approach for these multi-unit auctions. (Hint: what if for each i, B_i/v_i is a modestly large number but still far smaller than m?)

Exercise 40

Consider a multi-unit auction where bidders have private valuations per unit *and* private budgets. We can still try to run the clinching auction, by asking bidders to report their budgets and running the mechanism with the reported budgets and valuations. Prove that this mechanism is not DSIC.

[Hint: Dobzinski, Lavi, and Nisan (2008) offer the follow intuition: "If bidder A slightly delays reporting a demand decrease, bidder B will pay as a result a slightly higher price for his acquired items, which reduces his future demand. In turn, the fact that bidder B now has a lower demand implies that bidder A pays a lower price for future items..."]

Exercise 41

Prove that the output of the TTCA is uniquely defined — that is, it is independent of which cycle is chosen in each iteration of the algorithm. [You should prove this directly, without relying on any facts about the core of the housing allocation problem.]

Lecture 10 Exercises

Exercise 42

Recall the matching-based kidney exchange mechanism from lecture. Prove that this mechanism is DSIC — no agent (i.e., patient-donor pair) can be made better off (i.e., go from unmatched to matched) by reporting a strict subset F_i of its true edge set E_i.

Exercise 43

Show that there are cases in which the Gale-Shapley algorithm runs for $\Omega(n^2)$ iterations before terminating (with a stable matching).

Exercise 44

Prove that the Gale-Shapley proposal algorithm is not DSIC for the right-hand side (the women) — i.e., there are cases in which a woman can submit a false total ordering over the men and receive a better mate than when reporting its true preferences.