CS269I: Exercise Set #1

Due by 11:59 PM on Wednesday, October 5, 2016

Instructions:

(1) You can work individually or in a pair. If you work in a pair, the two of you should submit a single write-up.

(2) Submission instructions: We are using Gradescope for the homework submissions. Go to www.gradescope.com to either login or create a new account. Use the course code 9P8K49 to register for CS269I. Only one person needs to submit the assignment. When submitting, please remember to add your partner's name (if any) in Gradescope.

(3) Please type your solutions if possible. We encourage you to use the LaTeX template provided on the course home page.

(4) Write convincingly but not excessively. You should be able to fit all of your solutions into two pages, if not less.

(5) Except where otherwise noted, you may refer to the course lecture notes and the specific supplementary readings listed on the course Web page *only*.

(6) You can discuss the exercises verbally at a high level with other groups. And of course, you are encouraged to contact the course staff (via Piazza or office hours) for additional help.

(7) If you discuss solution approaches with anyone outside of your group, you must list their names on the front page of your write-up.

(8) No late assignments will be accepted, but we will drop your lowest exercise set score.

Lecture 1 Exercises

Exercise 1

Recall the room assignment problem and serial dictatorship mechanisms.

Serial Dictatorship Mechanism

1. Each student submits a ranked list (with no limit on the number of entries).
2. The students are ordered in some way. (E.g., by draw numbers.)
3. The students are considered in order. When student i is considered, she receives her top-ranked option that is still available.

With n students, there are $n!$ different serial dictatorship mechanisms — one for each possible ordering of the students.

We saw in lecture that every serial dictatorship is strategyproof and always produces a Pareto optimal outcome. Can you think of any other mechanisms for the room assignment problem that have these two properties?

Exercise 2

Prove that a serial dictatorship is *group-strategyproof*, meaning that even when students can collude, honesty is the best policy. Formally, prove that: for every subset S of students, if some coordinated misreport of the subset's ranked lists makes a student of S strictly better off, then this misreport also makes at least one student of S strictly worse off.

Exercise 3

Here's a bad alternative to a serial dictatorship, which unfortunately was used to assign kids to elementary schools in a number of major cities for many years.

A Bad Mechanism for One-Sided Markets

1. Each student submits a ranked list (with no limit on the number of entries).
2. The students are ordered in some way. (E.g., by lottery numbers.)
3. The students are considered in this order. When student i is considered, if her top-ranked school is still available, then she is (permanently) assigned to that school. (Otherwise, she is not assigned in this phase.)
4. The still-unassigned students are considered in the same order as before. When student i is considered, if her second-ranked school is still available, then she is assigned to that school. (Otherwise, she is not assigned in this phase.)
5. And so on with the still-unassigned students' third choices, fourth choices, etc.

Discuss in detail what type of strategic behavior (i.e., gaming of the system) you would expect to see from the participants in this mechanism. Do you think that some types of students would tend to be harmed more than others by the flaws of this mechanism?

Exercise 4

Note added after assignment was due: the mechanism below is an incorrect description of the original NRMP proposal, resulting in this exercise being more trivial than intended.

We mentioned in lecture that the initially proposed mechanism for the National Resident Matching Program was overruled by a group of protesting medical school students in favor of the deferred acceptance algorithm. Here is the original proposal:

A Bad Mechanism for Two-Sided Markets

1. Each student submits a ranked list of hospitals, and each hospital a ranked list of students (with no limits on length).
2. In the first phase, whenever a student s ranked h first and vice versa, s is (permanently) assigned to h. (Note that all such matches can be accommodated.) Call these the "1-1 matches."
3. In the second phase, whenever s is currently unassigned, s ranked h second, and also h ranked s first, then s is assigned to h. (Again, these can all be accommodated.) Call these the "2-1 matches."
4. In the third phase, the mechanism makes as many "2-2 matches" as possible. (Where s is unassigned and ranked h second, and h ranked s second.) In this phase, it is possible that a 2-2 match cannot be completed due to the hospital h being already full.

5. Subsequent phases consider 3-1 matches, then 3-2 matches, then 3-3 matches, then 4-1 matches, and so on.

Why do you think the medical students rejected this proposal? What were they worried about?

Lecture 2 Exercises

Exercise 5

Recall from lecture the stable matching problem, and that a *matching* is an assignment of students to hospitals. (If the capacity of hospital h is c_h, at most c_h students should be assigned to h.) A matching is *Pareto optimal* if every other matching that makes someone better off (i.e., matches them to a preferred student/hospital) also makes someone else worse off. Is every stable matching also Pareto optimal? Provide a proof or an explicit counterexample.

Exercise 6

Is every Pareto optimal matching also stable? Provide a proof or an explicit counterexample.

Exercise 7

Show by example that the deferred acceptance algorithm is not strategyproof for the hospitals. That is, exhibit a stable matching problem (students, hospitals, and their true ranked lists), a hospital h, and an untruthful preference list for h, such that h is strictly better off in the deferred acceptance algorithm by submitting the untruthful list than the truthful one. (Assume that everyone other than h reports their true lists.)