• Which auction formats are robust to shill bidding?

• Why are Dutch auctions so popular in Web3?

• What happens if you change the credible auctions model to have public rather than private communication?

A new paper with Andrew Komo and @skominers answers all these questions 👇 (1/19)

Shill-Proof Auctions*

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The paper lays out a theory of *shill-proof* auctions (2/19)

Abstract

In a single-item auction, a duplicitous seller may masquerade as one or more bidders in order to manipulate the clearing price. This paper characterizes auction formats that are *shill-proof*: a profit-maximizing seller has no incentive to submit any shill bids. We distinguish between *strong* shill-proofness, in which a seller with full knowledge of bidders' valuations can never profit from shilling, and *weak* shill-proofness, which requires only that the expected equilibrium profit from shilling is nonpositive. The Dutch auction (with suitable reserve) is the unique optimal and strongly shill-proof auction. Moreover, the Dutch auction (with no reserve) is the unique prior-independent auction that is both efficient and weakly shill-proof. While there are a multiplicity of strategy-proof, weakly shill-proof, and optimal auctions; any optimal auction can satisfy only two properties in the set {static, strategy-proof, weakly shill-proof}.

A *shill bid* is a bid submitted by the seller in order to manipulate the selling price of an auction. Shill bids have been around forever (e.g., the major art auction houses explicitly reserve the right to use them) and appear to be particularly common in online auctions (3/19)

Auction theory typically punts on dealing with seller deviations like shill bids via appeal to ummodeled concepts such as "rule of law." To what extent can these deviations instead be disincentivized through an auction's design? (4/19)

But wait: For private-value auctions, isn't a shill bid the same thing as a reserve price? The answer depends on when the seller has an opportunity to shill and the information available to them at that time (5/19)

Example: single-item auction, iid bidder valuations drawn from a regular distribution. In an English (ascending) auction with an optimal reserve price, a seller without extra knowledge of bidders' valuations has no incentive to shill (6/19)

If the seller *did* know bidders' precise valuations, they would want to shill (i.e., prolong the auction) up until the price is just below the highest bidder valuation (7/19)

Translating this to our terminology, the English auction (with optimal reserve) is *weakly shill-proof* but is not *strongly shill-proof* (8/19)

Could any auction be strongly shill-proof? Sure, in a Dutch (descending) auction, any shill bid by the seller terminates the auction immediately, leaving the seller holding the item and earning zero revenue (9/19)

Given these examples, anyone who knows us could guess the questions that we inevitably got obsessed with: (10/19)

- Which auction formats are "strongly shill-proof" in the sense of the Dutch auction, i.e., with shill bidding being unprofitable even with full knowledge of bidders' realized valuations?
- Which auction formats are "weakly shill-proof" in the sense of the English auction (with bidders' valuations drawn i.i.d. from a regular distribution and an optimally chosen reserve price), i.e., with shill bidding being unprofitable in expectation at equilibrium?
- To what extent are strong and weak shill-proofness compatible with other desirable properties such as optimality, efficiency, strategy-proofness, and sealed-bid implementation?

The answers can be visualized as a 4-D array, projected here as four 2 by 2 tables: (11/19)

	Static	Not Static
Strategy-Proof	Impossible (Theorem 5.2)	Ascending, Screening Auction (Theorem 4.7)
Not Strategy-Proof	First-Price Auction (Example 2.7)	Dutch Auction (Theorem 3.5)

(a) Weakly shill-proof and optimal auctions

	Static	Not Static
Strategy-Proof	Impossible (Theorem 5.2)	Not Robustly (Theorem 3.9)
Not Strategy-Proof	Not Robustly (Theorem 3.9)	Dutch Auction (Robustly Unique, Theorem 3.9)

(b) Weakly shill-proof and efficient auctions

	Static	Not Static
Strategy-Proof	Impossible (Theorem 3.5)	Impossible (Theorem 3.5)
Not Strategy-Proof	Impossible (Theorem 3.5)	Dutch Auction (Unique, Theorem 3.5)

(c) Strongly shill-proof and optimal auctions

	Static	Not Static
Strategy-Proof	Impossible (Theorem 3.9)	Impossible (Theorem 3.9)
Not Strategy-Proof	Impossible (Theorem 3.9)	Dutch Auction (Unique, Theorem 3.9)

(d) Strongly shill-proof and efficient auctions

We focus on "public" auctions in which every bidder's action is publicly observable (motivated by typical English/Dutch auctions, and typical Web3 auctions); cf., the private channels communication model in credible auctions @akbarpour_ @ShengwuLi (12/19)

Result 1 (of 4): the Dutch auction (with an optimal reserve) is the unique auction that is strongly shill-proof and optimal (i.e., revenue-maximizing); thus, such auctions must be iterative (with possibly many rounds) and cannot be strategy-proof (13/19)

Theorem 3.5. *A public and optimal auction is strongly shill-proof if and only if it is the testing Dutch auction with reserve price ρ^*.*

The rest of the results are about weakly shill-proof auctions (a substantially richer design space, as the English auction example makes clear) (14/19)

Result 2: if you want an auction that is robustly efficient (item goes to bidder with highest valuation, auction format not overly tailored to valuation distribution), you're still more or less forced to use a Dutch auction (15/19)

Theorem 3.9. *For every public and efficient auction that is not a semi-Dutch auction with cutoff ρ^*, there exists a regular value distribution with optimal reserve ρ^* under which the auction is not weakly shill-proof.*

Result 3: if you want an optimal auction that is also strategyproof, not only can you use an English auction (as in the example), but you can also use a "compressed" version that replaces the later rounds with a (one-shot) Vickrey auction (16/19)

Theorem 4.7. *If the value distribution is a discrete MHR distribution F, then for all*

$$v^Y \geqslant F^{-1}\left(F\left(\rho^*\right)+\max_{1\leqslant n<N}\left\{\left(\max\left\{1-\frac{\rho^*}{\rho^*+2\overline{\Delta}}\left(\frac{f(\rho^*)}{1-F(\rho^*)}\right)^n,0\right\}\right)^{1/n}\right\}\right), \quad (3)$$

the ascending, screening auction with screening level v^Y is a weakly shill-proof, strategy-proof, and optimal auction.

Result 4: the number of rounds cannot be compressed to 1. In general, impossible to get weakly shill-proof+optimal+strategyproof+one-shot; for optimal auctions, can pick 2 and only 2 of {weakly shill-proof,strategyproof,one-shot} (17/19)

Theorem 5.2. *There exists no one-shot, optimal auction that is mildly strategy-proof and weakly shill-proof.*

Credible auction buffs may also enjoy Section 5.2, which shows a sense in which strong shill-proofness => credibility => weak shill-proofness (18/19)

Full paper is here, comments and questions are welcome!: https://arxiv.org/abs/2404.00475 (19/19)