CS261: Exercise Set #3

For the week of January 18–22, 2016

Instructions:

(1) *Do not turn anything in.*

(2) The course staff is happy to discuss the solutions of these exercises with you in office hours or on Piazza.

(3) While these exercises are certainly not trivial, you should be able to complete them on your own (perhaps after consulting with the course staff or a friend for hints).

Exercise 11

Recall that in the maximum-weight bipartite matching problem, the input is a bipartite graph $G = (V \cup W, E)$ with a nonnegative weight w_e per edge, and the goal is to compute a matching M that maximizes $\sum_{e \in M} w_e$.

In the minimum-cost perfect bipartite matching problem, the input is a bipartite graph $G = (V \cup W, E)$ such that $|V| = |W|$ and G contains a perfect matching, and a nonnegative cost c_e per edge, and the goal is to compute a perfect matching M that minimizes $\sum_{e \in M} c_e$.

Give a linear-time reduction from the former problem to the latter problem.

Exercise 12

Suppose you are given an undirected bipartite graph $G = (V \cup W, E)$ and a positive integer b_v for every vertex $v \in V \cup W$. A *b-matching* is a subset $M \subseteq E$ of edges such that each vertex v is incident to at most b_v edges of M. (The standard bipartite matching problem corresponds to the case where $b_v = 1$ for every $v \in V \cup W$.)

Prove that the problem of computing a maximum-cardinality bipartite b-matching reduces to the problem of computing a (standard) maximum-cardinality bipartite matching in a bigger graph. Your reduction should run in time polynomial in the size of G and in $\max_{v \in V \cup W} b_v$.

Exercise 13

A graph is *d-regular* if every vertex has d incident edges. Prove that every d-regular bipartite graph is the union of d perfect matchings. Does the same statement hold for d-regular non-bipartite graphs?

[Hint: Hall's theorem.]

Exercise 14

Prove that the minimum-cost perfect bipartite matching problem reduces, in linear time, to the minimum-cost flow problem defined in Lecture #6.

Exercise 15

In the *edge cover* problem, the input is a graph $G = (V, E)$ (not necessarily bipartite) with no isolated vertices, and the goal is to compute a minimum-cardinality subset $F \subseteq E$ of edges such every vertex $v \in V$ is the endpoint of at least one edge in F. Prove that this problem reduces to the maximum-cardinality (non-bipartite) matching problem.